

# $\Sigma$ SCiM 2024



## BOOK OF ABSTRACTS

**15<sup>th</sup>** European Symposium on Computational  
Intelligence and Mathematics  
May 12<sup>th</sup> – 15<sup>th</sup>, 2023 • Krakow, Poland

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# Book of abstracts of ESCIM 2024

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Roberto G. Aragón, Fernando Chacón-Gómez, María Eugenia Cornejo,  
Janusz Kacprzyk, David Lobo, Francisco J. Ocaña-Alcázar,  
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Program of the  
15<sup>th</sup> European Symposium  
on Computational Intelligence and Mathematics  
including Workshop DigForASP  
May 12<sup>th</sup> - 15<sup>th</sup>, 2024. Krakow, Poland



MONDAY 13 <sup>th</sup>	
Location: Matejko Hotel	
8:30	Open Registration Desk
9:00–9:30	Inauguration
9:30–10:20	<b>Keynote Speaker - Janusz Kacprzyk</b> Title: An approach to Consensus and dissensus driven decision making under fuzziness Chairperson: Jesús Medina
10:20–11:20	<b>Session S1.</b> Chairperson: Janusz Kacprzyk
	<i>A note on aggregation of T-fuzzy subgroups</i> Andreja Tepavcevic, Ivana Štajner-Papuga
	<i>Independent subcontexts in fuzzy formal concept analysis</i> Roberto G. Aragón, Jesús Medina and Eloísa Ramírez-Poussa
	<i>Two sufficient conditions for having a maximal solution to fuzzy relational equations</i> Vanja R. Stepanović
11:20–11:40	Coffee break
11:40–13:20	<b>Special Session: Rough Sets and Information Granulation</b> Chairperson: Jarosław Was
	<i>Towards compatible subcontext by means of classical attribute reduction</i> María José Benítez-Caballero, Jesús Medina
	<i>Extension of The Time Dependent Travelling Salesman Problem Model with Interval-Valued Fuzzy Soft Sets with Arithmetic Mean Method</i> AlMahasneh Ruba, László T. Kóczy
	<i>Representation of distances between objects using weighted distance graph with rough set membership: application to cosmic rays dataset annotation</i> Tomasz Hachaj, Marcin Piekarczyk, Jarosław Was
	<i>Typology of Granular Sets based on Granular Spectrum of Covering Rough Sets</i> Piotr Wasilewski, Dominik Ślęzak
	<i>Ranking Bipolarity in Partial Approximation Spaces</i> Zoltán Ernő Csajbók
13:20–14:40	Lunch

14:40–15:30	<p><b>Keynote Speaker - Radu-Emil Precup</b>  Title: Applications of metaheuristic algorithms to fuzzy control and model building, learning-based control, and mobile robot navigation.  Chairperson: László T. Kóczy.</p>
15:30–16:00	Coffee break
16:00–16:40	<p><b>Regular Session S2.</b>  Chairperson: Roberto G. Aragón</p>
	<p><i>On aggregation operators and utility functions with truncation property</i>  Dragan Jočić, Ivana Štajner-Papuga</p>
	<p><i>Categories of L-fuzzy morphological operators on L-fuzzy groups and LR-fuzzy homomorphisms</i>  Ksenija Varfolomejeva, Alexander P. Šostak</p>
16:40–18:00	<p><b>Special Session: Decision and optimization model applied to logistics and transport.</b>  Part 1.  Chairperson: Julio Alberto López-Gómez</p>
	<p><i>Message Passing Graph Neural Network for Seeds Classification</i>  Piotr Moszkowicz, Piotr A. Kowalski, Tomasz Bold</p>
	<p><i>Fuzzy Rule Systems design to model the decision method of a passenger profile</i>  David Muñoz Valero, Enrique Adrián Villarrubia Martín, Julio Alberto López-Gómez, Juan Moreno-García</p>
	<p><i>An interval-valued fuzzy soft sets based decision support model for route optimization</i>  Boldizsár Tüü-Szabó, Ruba AlMahasneh, László T. Kóczy</p>
	<p><i>Railway Capacity Allocation in Liberalized Markets: First Approach for an Artificial Intelligence-Based Computational Model</i>  David Muñoz Valero, Enrique Adrián Villarrubia Martín, Julio Alberto López-Gómez, Juan Moreno-García</p>
19:30	<b>Welcome reception</b>

<b>TUESDAY 14<sup>th</sup></b> <b>Workshop DigForASP</b> <b>Location: Matejko Hotel</b>	
9:00–9:50	<b>Keynote Speaker - Viviana Mascardi</b> Title: Logic Programming and Legal Reasoning: the Past and the Future Chairperson: Stefania Costantini
9:50–10:50	<b>Workshop DigForASP. Session 1</b> Chairperson: Viviana Mascardi
	<i>Semi-automatic knowledge representation and reasoning on vague crime concepts</i> Manuele Dozzi, Talissa Dreossi, Luca Baron, Federico Costantini, Agostino Dovier, Andrea Formisano
	<i>Empowering Emotional Behavior Trees with Neural Computation for Digital Forensic</i> Stefania Costantini, Giovanni De Gasperis, Pierangelo Dell'Acqua, Andrea Rafanelli
	<i>Legal and technical challenges of AI in the field of Criminal investigations</i> Dévika Pérez-Medina, Nicolás Madrid, Piotr A. Kowalski
10:50–11:10	Coffee break
11:10–12:00	<b>Workshop DigForASP. Session 2</b> Chairperson: Manuel Ojeda-Aciego
	<i>Fingerprint Revolution: Unleashing the Potential of Modified Bacterial Memetic Evolution for a Paradigm Shift in Fingerprint Recognition and Optimization</i> Ahmad Momani, László T. Kóczy
	<i>Comparison of text similarity techniques for power of attorney clauses for Polish banks</i> Karolina Wadowska and Piotr A. Kowalski
	<i>Using Human-Computer Interaction Data to Improve Authentication Reliability in High-stake Electronic Assessments</i> Danilo Strugarevic, Nikola Gligorijevic, Goran Simic, Aleksandar Jevremovic
12:20–13:40	<b>Regular Session 3</b> Chairperson: Stefania Costantini
	<i>Belief Change: Axiomatic Characterization of KM-Erasure</i> Eduardo Fermé
	<i>Stable Reasoning, ASP and the Interrogative Model of Inquiry</i> David Pearce, Agustín Valverde
	<i>Hypergraph logic program representation versus stratified programs</i> David Lobo, Jesús Medina, José Ramón Portillo, José Antonio Torné-Zambrano
	<i>A preliminary taxonomy of explanations in problem solving</i> Pedro Cabalar, Esra Erdem, Muge Fidan, Brais Muñiz
	<i>Efficiency of decision rule sets in classification problems</i> Fernando Chacón-Gómez, María Eugenia Cornejo, Jesús Medina
13:40–15:00	Lunch
15:00–16:00	<b>Round table. Next steps DigForASP</b>
17:00–19:30	<b>Krakow tour</b>
20:00	<b>Gala dinner</b>

WEDNESDAY 15 <sup>th</sup>	
Location: Matejko Hotel	
10:00–11:00	<b>Keynote Speaker - Piotr A. Kowalski</b> Title: Sensitivity Analysis as a Method for Explaining AI (XAI) in the Artificial Neural Networks Domain Chairperson: M. Eugenia Cornejo
11:00–11:20	Coffee break
11:20–12:00	<b>Special Session: Decision and optimization model applied to logistics and transport. Part 2</b> Chairperson: Julio Alberto López-Gómez
	<i>Enhancement of Discrete Bacterial Memetic Evolutionary Algorithm for solving The Travelling repairman Problem</i> Ali Jawad Ibada, Boldizsár Tüü-Szabó, László T. Kóczy
	<i>Simulated Annealing and Bacterial Foraging for Probabilistic Neural Network parameters adjustment</i> Szymon Kucharczyk, Piotr A. Kowalski
12:00–13:30	<b>Special Session: Recent trends in knowledge representation and modelling</b> Chairperson: Manuel Ojeda-Hernández
	<i>Extreme Learning Machine as a New Learning Paradigm: Pros and Cons</i> Irina Perfilieva, Nicolás Madrid, Manuel Ojeda-Aciego, Piotr Artiemjew, Agnieszka Niemczynowicz
	<i>Inheritance of completeness between systems of strong and weak implications</i> Francisco Pérez-Gámez, Pablo Cordero, Carlos Bejines-López, Manuel Ojeda-Hernández, Domingo López-Rodríguez
	<i>Connections between attribute implications in heterogeneous formal contexts and GUHA association rules</i> L'ubomír Antoni, Peter Eliaš, Ján Guniš, Dominika Kotlárová, Stanislav Krajčí, Ondrej Krídlo, L'ubomír Šnajder
	<i>Migrative properties for triangular conorms and fuzzy implications</i> M. Eugenia Cornejo, Jesús Medina, Francisco José Ocaña
	<i>Masonry strength assessment based on Fuzzy signature model</i> András Kaszás, András Dormány, Vanda Pomezanski, Zoltán Orbán, László T. Kóczy
13:30–15:00	<b>Closing Session</b>



*Keynote speech:*

## An approach to Consensus and dissensus driven decision making under fuzziness

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**Abstract:** We advocate a new way of decision making under fuzzy preferences and fuzzy majorities the essence of which is that the consensus driven paradigm, which is very often employed, is not followed as it can often prohibit reaching innovative decisions.

First, we briefly review main developments in the broadly perceived group decision making and indicate the main general approaches, namely via: 1) Unanimous decisions when all agents agree without reservations; 2) Consensus in which each agent agrees to give his/her consent to the decision reached, even if it would not be perfect, but acceptable, and is open to modify his or her testimonies; 3) Majority Rule when, e.g., more than a half, at least  $2/3$ , etc, of the group votes in favor; 4) Expert in which a special agent, an expert, is chosen for running the decision making process; 5) Executive in which a special high level agent makes the decision with little or none involvement of other group members; 6) Default in which a decision is made as needed, without any analysis.

We analyze some new directions in social sciences, cognitive science, psychology, decision theory, etc. in which there is an explicit critique of consensus as a viable and effective and efficient way of making group decisions. Basically, the argument is that “consensus is the quickest way to kill innovation”. This argument is raised by many authors, and there are some attempts to use some more formal analyses.

We also use some results of the so called entrepreneurial action theory which also mentions that the most interesting forms of entrepreneurship involve ideas that contradict prevailing wisdom, and opinions. These behaviors may be irrational and seem impossible but may lead to innovative, even revolutionary outcomes.

We propose a new model of dissensus driven group decision making under fuzzy preferences and fuzzy majorities. We redefine the solution concepts along the line of the fuzzy cores and fuzzy consensus winners. We mention an examples.

# A note on aggregation of $T$ -fuzzy subgroups

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**Abstract:** This paper aims to provide an answer to the question of the preservation of fuzzy subgroup structure for some special classes of aggregation operators. Both binary and  $n$ -ary aggregation of fuzzy subgroups are considered. The main contribution of the approach proposed in this paper is the development of a new method for  $n$ -ary aggregation applicable to fuzzy subgroups regardless of the cardinality, based on  $n$ -ary aggregation operators generated by a family of binary aggregation operators.

**Keywords:** Fuzzy subgroups · Aggregation operator

**Acknowledgement:** This research was supported by the Science Fund of the Republic of Serbia, # Grant no 6565, Advanced Techniques of Mathematical Aggregation and Approximative Equations Solving in Digital Operational Research- AT-MATADOR.

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# Independent subcontexts in fuzzy formal concept analysis

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**Abstract:** Independent subcontext is a fundamental notion for different purposes, such as in factorization of contexts in both the Boolean and fuzzy case. Although the notion can be intuitive, a mathematical definition is required. This paper formalizes this notion and provides some properties of the independent subcontexts of an  $L$ -fuzzy context.

**Keywords:** Formal concept analysis · factorization · independent subcontext.

**Acknowledgement:** Partially supported by the project PID2019-108991GB-I00 funded by MICIU/AEI/10.13039/501100011033, the project PID2022-137620NB-I00 funded by MICIU/AEI/10.13039/501100011033 and FEDER, UE, by the grant TED2021-129748B-I00 funded by MCIN/AEI/10.13039/501100011033 and European Union NextGenerationEU/PRTR, and by the project PR2023-009 funded by the University of Cádiz.

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# Two sufficient conditions for having a maximal solution to fuzzy relational equations

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**Abstract:** The existence of a maximal solution to some typical fuzzy relational equations is proved in the paper, in case when the codomain lattice is complete and meet-continuous, which means that the infimum commutes with the supremum of chains. These results extend the existing results, which are mostly limited to the existence of a maximal solution to some typical fuzzy set and fuzzy relational inequations. In order to prove that the same holds for the corresponding relational equations, it is proved that the property of meet-continuity in the codomain lattice implies another property in the lattice of fuzzy relations, namely that the composition of fuzzy relations, defined as usual, commutes with the supremum of chains. This condition can also be taken instead of the meet-continuity of the codomain lattice as another sufficient condition for the existence of a maximal solution to the equations considered here. Two examples are given, which prove that these conditions do not imply the existence of the greatest solution to some of the considered equations.

**Keywords:** Complete lattice · Meet-continuous lattice · Fuzzy relational equations · Maximal solution

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# Towards compatible subcontext by means of classical attribute reduction

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**Abstract:** Reducing a database is one of the main goals of the theories developed for the management of information in databases, such as Formal Concept Analysis (FCA) and Rough Set Theory (RST). On the other hand, this reduction should preserve some properties from the original database. In FCA, for example, it is recommendable to obtain compatible subcontexts. This paper introduces sufficient conditions to obtain compatible subcontexts from attribute reduction theory in FCA.

**Keywords:** Formal Concept Analysis · Attribute Reduction · Reduct · Compatible Subcontext

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# Extension of The Time Dependent Travelling Salesman Problem Model with Interval-Valued Fuzzy Soft Sets with Arithmetic Mean Method

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**Abstract:** The Traveling Salesman Problem (TSP) is an extensively studied NP-hard graph search problem. Many researchers pursued the most efficient and practical solutions, by applying various techniques to find the optimum or semi optimum solution (the one with least cost). There are numerous practical extensions and modifications of the original problem, such as The Time Dependent Traveling Salesman Problem (TD TSP). Indeed, the TD TSP was towards more realistic assessment of the traffic conditions of the original TSP. The edges between nodes are assigned different costs (weights), whether they are traveled during the rush hour periods or if they crossed the traffic jam regions (such as city centers). In the classic TD TSP, the edges are assigned higher costs using concrete numbers, which might be looked at as a limitation; because those jam factors are non-deterministic and better be represented by fuzzy numbers. In this paper we introduce a more realistic novel fuzzy-based extension, the IVFSSTD TSP (Interval-Valued Fuzzy Soft Set for the Time Dependent Traveling Salesman Problem). Our core concept employs interval-valued fuzzy soft sets on the costs between nodes to realistically quantify the traffic jam regions, and the rush hours periods effects on any tour, then we use the arithmetic mean operator to take in account all factors affecting an edge simultaneously, which lead to less information loss and more adequate representation for the jam factors. Since the interval-valued fuzzy soft sets are generalization of the original fuzzy sets, which has the ability to simulate uncertain road conditions more efficiently than concrete numbers, then our approach can be considered a useful extension and a practical alternative model of the original abstract problem.

**Keywords:** Rush hours · Jam regions · Interval-valued fuzzy soft sets · Time Dependent Traveling Salesman Problem.

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# Representation of distances between objects using weighted distance graph with rough set membership: application to cosmic rays dataset annotation

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**Abstract:** In this work, we define a weighted distance graph with rough set membership [7,8,9] and its application to cosmic rays dataset annotation. The weighted distance graph we introduce is a weighted undirected graph whose vertices represent objects and the weights of the edges connecting the objects have weights that are a pair. The first element of the pair equals the distance between the objects that the edge connects. The second element of the pair is equal to the type of membership approximation in the sense of Rough set (lower or upper approximation). The objects can be, for example, m-dimensional vectors, between which it is possible to calculate the distance using, for example, the Euclidean metric. Formally, we define such a graph as a triple:

$$\mathbb{G}_k = (V, E, w) \quad (1)$$

where  $V$  is a set of  $l$  vertices (each vertex represents object defined by by m-dimensional vector),  $E$  is a set of undirected edges (due to the fact that distance function is symmetric) that links nodes,  $w$  is a function mapping edges to their values.

$$w : E \rightarrow (\mathbb{R}_+ \cup \{0\}, \{lower, upper\}) \quad (2)$$

Value  $\mathbb{R}_+ \cup \{0\}$  is calculated using the  $f$  metric defined on  $(N \times N)$ ,  $N$  is a set of all  $l$  objects.

The letter  $k$  in the notation  $\mathbb{G}_k$  is directly related to the way this graph is generated. In order to check whether two vertices belonging to  $V$  are connected by an edge, for each vertex of the graph it is necessary to find the  $k$  nearest objects to it with respect to the  $f$  metric. Only these  $k$  closest objects will be connected by edges. For two vertices  $v_1, v_2 \in V$  one of two situations may occur:

$$p^{v_1} = \min_k f(\{v_1\} \times (V - \{v_1\})) \quad (3)$$

$$p^{v_2} = \min_k f(\{v_2\} \times (V - \{v_2\})) \quad (4)$$

$$\begin{cases} f(v_1, v_2) \in p^{v_1} \cap f(v_1, v_2) \in p^{v_2} \rightarrow lower \\ f(v_1, v_2) \in p^{v_1} \cap f(v_1, v_2) \notin p^{v_2} \rightarrow upper \end{cases} \quad (5)$$

where  $\min_k f(\{v_1\} \times (N - \{v_1\}))$  is the set of  $k$  smallest distances between object  $v_1$  and all other objects belonging to  $V$ . (5) should be interpreted as follows: if object  $v_1$  is among  $k$  nearest objects to  $v_2$  and object  $v_2$  is among  $k$  nearest objects to  $v_1$  then

both objects are in the lower approximation of rough set. If object  $v_1$  is among  $k$  closest objects to  $v_2$  and object  $v_2$  is not among  $k$  closest objects to  $v_1$  then both objects are in the upper approximation of rough set. The information whether the objects are in lower or upper approximation is added to the weight of the edge between these vertices. Graph  $\mathbb{G}_k$  does not have self-loops.

If  $k = l$ , then graph  $\mathbb{G}_k$  is a complete graph and all weights between vertices have lower approximation.

The  $\mathbb{G}_k$  graph can be a useful tool, for example, for accelerating the semi-manual annotation of various datasets. In our case, we used it for annotation of cosmic rays dataset [2], which was collected by the Cosmic Ray Extremely Distributed Observatory (CREDO) project [6]. The dataset is composed of images with a resolution of  $60 \times 60$  pixels [1] acquired with CMOS sensors [3]. Using principal component analysis (PCA), we performed dimensionality reduction leaving the 62 most significant dimensions. This corresponds to more than 95 percent of the variance in the dataset [5]. We then constructed a  $\mathbb{G}_k$  graph for  $k = 5$ . We sorted the  $V$  vertices according to descending vertex degree. Each vertex was also given a status, whether it had already been visited or not. Then, starting from the vertex with the highest degree, we performed object annotation by assigning each object to one of four classes [4]. A class is an interpretation of the type of particle visible in the image based on the morphology of the trace left by it. After annotating a single particle  $v_i$ , we also annotated all particles  $v_j$  that were connected by an edge to  $v_i$  assigning them the same class as the particle  $v_i$ . If the edge had a weight denoting rough set lower approximation, we mark the vertex  $v_j$  as visited. From the list of edges connected to  $v_j$ , we removed all edges connecting  $v_j$  to vertices that already had a class assigned. We also removed all edges connecting  $v_i$  to other vertices. The above process was repeated until the degree of all  $V$  rows were equal to zero. The next step of the algorithm was an additional semi-manual validation of objects that had not been visited so far, and obtained automatic annotation by the fact of belonging to the  $k$  neighborhood of the closest objects. The use of rough sets approximation made it possible to eliminate objects that were close to each other from the process. This approach is very similar to density-based clustering except that it does not require a distance threshold. Currently there is no proposed method that indicates the optimal  $k$  for a given problem. The value of  $k$  is therefore estimated by an expert. In the case of the CREDO set, we know that the clusters in our embedding are not concentric. Therefore, in our application, we chose a small value of  $k$  relative to the size of the dataset. Thanks to the application of the discussed algorithm, we fully annotated the considered CREDO dataset. This achievement was very important, because we could use the set prepared in this way for further supervised classifier training.

**Keywords:** Weighted distance graph · Rough set · Data annotation · Cosmic rays dataset

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# Typology of Granular Sets based on Granular Spectrum of Covering Rough Sets

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**Abstract:** In [8] we introduced granular spectrum of covering rough sets (see Fig. 1). In this paper we propose a typology of granular sets which is based on granular spectrum of covering rough sets. This typology is proposed by analogy to the typology of classical rough sets proposed by Z. Pawlak.

Rough sets were proposed [3], [4] (see also: [5]) to represent vague concepts and to deal with incomplete information [1], [2], [7]. Lower and upper approximations of any set  $X \subseteq U$  for the approximation space  $(U, R)$  and  $R$  an equivalence relation of  $U$ :

$$R_*(X) := \bigcup \{Y \in U_{/R} : Y \subseteq X\} \quad R^*(X) := \bigcup \{Y \in U_{/R} : Y \cap X \neq \emptyset\},$$

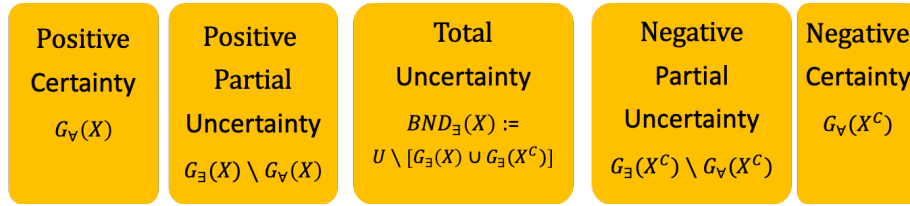
were interpreted by Pawlak [4] as *conceptual measures* of a set  $X$  by analogy to *inner Jordan measure* and *outer Jordan measure* in space  $\mathbb{R}^n$ . On the basis of this interpretation Pawlak suggested a kind of typology of rough sets in which sets were catalogued whether their lower or upper approximations or their boundaries are empty sets, essential subsets of the space  $U$  or equal to the space  $U$ . For example, in such typology for the selector  $S$  of the family  $U_{/R}$ :  $R_*(S) = \emptyset$  and  $R^*(S) = U$ . Actually, diagrams used in the rough set theory for representation of lower and upper approximations of sets are derived from the inner Jordan measure and the outer Jordan measure of the plane  $\mathbb{R}^2$ .

The granular spectrum of the object space  $U$  is based on arbitrary coverings of the object space in [8]. These approximation operators were introduced in [10]:

$$G_{\forall}(X) := \{a \in U : \forall A \in Gr(a) A \subseteq X\} \quad G^{\exists}(X) := \{a \in U : \exists A \in Gr(a) A \cap X \neq \emptyset\}.$$

$$G_{\exists}(X) := \{a \in U : \exists A \in Gr(a) A \subseteq X\} \quad G^{\forall}(X) := \{a \in U : \forall A \in Gr(a) A \cap X \neq \emptyset\}.$$

where  $Gr(a) := \{A \in Gr(U) : a \in A\}$  and  $Gr(U)$  is a family of granules covering the space  $U$ . Operators  $G_{\forall}$  and  $G^{\exists}(X)$  were investigated in [6] where the operator  $G^{\exists}$  was defined in a new equivalent form using biting procedure. One can note that operators  $G_{\forall}$  and  $G^{\exists}(X)$  and operators  $G_{\exists}$  and  $G^{\forall}$  are dual, i.e. for example  $G_{\forall}(X)^c = G^{\exists}(X^c)$ .



**Fig. 1.** Granular spectrum of space  $(U, Gr(U))$  determined by subset  $X \subseteq U$ . [8]

Operators  $G_v$ ,  $G^\exists(X)$ ,  $G_\exists$  and  $G^v$  are related to each other in the following way:

$$G_v(X) \subseteq G_\exists(X) \subseteq X \subseteq G^\exists(X) \subseteq G^v(X).$$

when approximating any set  $X \subseteq U$ . All five regions presented in Fig. 1 are pairwise disjoint and their union covers the space  $U$ . However in general this granular spectrum does not have to be a partition of the space  $U$  since all five regions are not necessary non-empty. This gives the reason for introducing a typology of granular sets analogous to typology of rough sets proposed by Pawlak. One can note that not all possible cases  $2^5 = 32$  are realized. For example, there are granular spaces  $(U, Gr(U))$  and sets such that *positive certainty* and *negative certainty* regions are empty making *total uncertainty* region equal to the whole space  $U$  and so making the remaining two regions also empty.

**Keywords:** rough sets · granular sets · coverings · covering-based rough sets · granular spectrum of covering rough sets · vague concepts · incomplete information

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# Ranking Bipolarity in Partial Approximation Spaces

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**Abstract:** Some phenomena have two distinct aspects, “positive” and “negative”. Suppose that a dataset is available to describe these aspects. We propose a model to decide which one has the best evaluate value. First, based on the dataset, we offer a new partial approximation space, which serves a better set approximation than Pawlak’s approximation space. Then, assigning two intervals to the two aspects, we evaluate them with the possibility degree formula, an effective tool for Multi-Attribute Decision Making (MADM) methods under uncertain environments. An example is used to illustrate how the theoretical construction works in practice. The proposed model can be applied to any two distinct entities and, even more, can be generalised to more than two separate entities.

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*Keynote speech:*

## Applications of metaheuristic algorithms to fuzzy control and model building, learning-based control, and mobile robot navigation

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**Abstract:** An optimization problem finds the best (i.e., optimal) solution among all feasible solutions. An optimization problem consists of two key components: the objective function and the constraints, which are optional. The objective function evaluates and compares solutions in the context of all feasible solutions by computing the desired quantity to be minimized or maximized. Constraints can be added to limit the possible values for the variables of the objective function and possibly to link these variables. The optimization algorithms find the solutions to the optimization problems (i.e., the optimal solutions) by trying variations of the initial solution and using the information gained to improve the solution. This solution finding can also be considered as learning, which is a popular topic nowadays. The complexity of classical algorithms is very high, which requires rather large amount of computation. Therefore, alternative algorithms with lower complexity are appreciated. Metaheuristic algorithms for finding optimal solutions have become very popular because they are much better in terms of efficiency and complexity than classical algorithms. This presentation highlights some of the results obtained by the Process Control Group of the Politehnica University of Timisoara, Romania. The presentation will focus on representative applications implemented in our labs, with real-time validation against experimental results. The results highlighted here include various laboratory equipment such as pendulum crane systems, multi-tank systems, servo systems, twin rotor aerodynamic systems, magnetic levitation systems, anti-lock braking systems, mobile robots, magnetic levitation systems, active mass damper systems, and shape memory alloy systems. The scope of development of these metaheuristic algorithms is to solve optimization problems involving tuning of low-cost fuzzy controllers, tuning of fuzzy models, reinforcement-based control in various schemes including adaptive ones, and solving optimization problems specific to mobile robot navigation.

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# On aggregation operators and utility functions with truncation property

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**Abstract:** Aggregation operators form a vast class of mathematical tools that are highly applicable in modelling decision-making processes. Of special interest are aggregation operators with an annihilator, i.e., with an absorbing element, since they can be applicable in modelling situations involving participants with the power of veto. This paper presents an overview of results that consider the conditional distributivity of such operators, namely  $T$ -uninorms and nullnorms, over  $t$ -conorms and an analysis of the role of obtained distributive pairs in constructions of utility functions with a truncation property.

**Keywords:** Aggregation operators · annihilator ·  $T$ -uninorms · nullnorms · conditional distributivity · utility function

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# Categories of $L$ -fuzzy morphological operators on $L$ -fuzzy groups and $LR$ -fuzzy homomorphisms

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**Abstract:** Given a completely distributive lattice  $L$ , we introduce two categories whose objects are  $L$ -powersets  $L^X$  of additive groups  $(X, +, 0)$  and the morphisms are respectively forward and backward operators  $R^\rightarrow : L^X \rightarrow L^Y$  and  $R^\leftarrow : L^Y \rightarrow L^X$  induced by  $L$ -relations  $R : X \times Y \rightarrow L$  which preserve the algebraic structure of the underlying groups  $(X, +_X, 0_X)$  and  $(Y, +_Y, 0_Y)$ . The objects of these categories serve as the field of action for operators of fuzzy erosion and dilation. We study some properties of these operators, in particular their behaviour under operators  $R^\rightarrow$  and  $R^\leftarrow$ .

**Keywords:** Completely distributive lattices ·  $L$ -relations · Categories of  $L$ -fuzzy groups · morphological operators on  $L$ -fuzzy groups

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# Message Passing Graph Neural Network for Seeds Classification

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**Abstract:** Within the field of High Energy Physics various machine learning techniques are being constantly researched. Due to upcoming HL-LHC (High Luminosity Large Hadron Collider) upgrade, which aims to increase probability of occurrence of rare physical phenomena by significant increase of number of collisions occurring each second, currently used algorithms in the area of charged particle reconstructions are not going to scale well enough. The data from tracking detectors if the LHC experiments can be represented in the form of a graph, where detected signals are represented as nodes and hypothetical connections between them are represented by edges. In particular the initial stage of track finding, the formation of seeds can be represented in that way. Within this paper we propose solution based on Graph Neural Network used to dramatically decrease number of seeds and therefore significantly reduce amount of calculations needed by downstream algorithms in order to reconstruct tracks. The numerical experiments were carried out with use a data set obtained from Monte Carlo simulation of the Open Data Detector. The initial results showed, that proposed solution based on Graph Neural Network has application potential.

**Keywords:** Graph Neural Networks · Clasification task · Reconstruction · Tracking · Large Hadron Collider · ATLAS · High Energy Physics

**Acknowledgement:** This work was partially supported by the program „Excellence initiative – research university” for the AGH University of Krakow and by Grant for Statutory Activity from the Faculty of Physics and Applied Computer Science of the AGH University of Krakow.

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# Fuzzy Rule Systems design to model the decision method of a passenger profile

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**Abstract:** The method by which a passenger purchases a train ticket is crucial for railway companies to provide appropriate offers, both to the passenger and to the infrastructure operator. In this paper we study how Fuzzy Rule Systems can be used to model the decision-making process of a train passenger when purchasing a ticket. In addition, two examples are presented that have allowed us to extract the first ideas for the design of an automatic method for the generation of fuzzy rule systems to support in the decision-making process of train passengers.

**Keywords:** Decision model · Fuzzy Rules Systems · Fuzzy Sets

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# An interval-valued fuzzy soft sets based decision support model for route optimization

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**Abstract:** Route optimization is an extensively studied NP-hard graph search problem. Many researchers applied numerous techniques to find the optimum or semi optimum solution (the one with least cost). There are many practical extensions and modifications of this problem applied using deterministic methods. However, traveling between nodes (locations) might encounter additional fuzzy cost (time) on the overall trip, whether they are traveled during the rush hour periods or if they crossed traffic regions (the city centers). Since, those factors are non-deterministic; it would be closer to reality to represent them using fuzzy numbers. In this paper, we propose a novel route optimization under road uncertainties using Interval-Valued Fuzzy Soft Sets. We use scoring technique to help determining the optimum route amongst all alternatives. Our novel approach can be looked at as a practical and closer to reality estimation for non-deterministic factors of the original abstract route optimization problem.

**Keywords:** routing optimization problem · interval-valued fuzzy soft sets · vehicle routing problem.

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# Railway Capacity Allocation in Liberalized Markets: First Approach for an Artificial Intelligence-Based Computational Model

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**Abstract:** In response to the European Union's push towards liberalized railway markets, this study examines the complexities of capacity allocation among various Railway Undertakings. The evolution of market structures is explored, from monopolistic to shared market structures, emphasizing Spain's railway market transition. Optimizing infrastructure utilization and resolving the challenges of slot distribution in a liberalized market are the main focus of the research. Building on previous works, limitations in existing capacity allocation models has been detected, and advanced solutions based on Artificial Intelligence are proposed. By integrating neural networks and metaheuristic algorithms, more complex and large-scale railway systems could be managed more effectively. This methodology explores alternative capacity allocation mechanisms for efficient train scheduling. The future scope of this research includes further development and comparative analysis to enhance railway network efficiency and sustainability.

**Keywords:** Capacity Allocation · Train Timetabling Problem · Liberalization · Metaheuristics · Artificial Neural Networks

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*Keynote speech:*

## Logic Programming and Legal Reasoning: the Past and the Future

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**Abstract:** Logic Programming is an extremely powerful tool for legal reasoning due to its declarative, rule-based nature, suitable for modelling in a natural and understandable way legal rules and constraints. In this talk, I will discuss the advantages and disadvantages of exploiting Logic Programming in the legal domain, and I will provide examples of use in different Logic Programming languages.

# Semi-automatic knowledge representation and reasoning on vague crime concepts

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**Abstract:** In this paper we address the formal representation of vagueness in the legal domain, focusing on a “use case” in the Italian criminal law, namely the distinction of two crimes, “snatching” and “robbery”. After a few epistemological clarifications on the concept of vagueness and a short premise on the legal background, we tackle the problem by adopting Answer Set Programming as modelling language. First, we encode the “static” law parts and then we enhance the encoding by learning from sentences. This is a first step for a legal reasoning system capable of evolving by doing a fully automatic learning from sentences.

**Keywords:** Legal Reasoning · Automated Reasoning · ASP · Criminal Law · Vagueness.

**Acknowledgement:** Research partially supported by Action COST CA17124 *Dig-ForASP*, by Inter-departmental projects on Artificial Intelligence and on Cross Border Digital Forensics (*Strategic Plan UniUD-22-25*), and by INdAM-GNCS project CUP E53C22001930001. Although the contribution is the result of a joint discussion among coauthors, Sections can be attributed as follows: 2 and 3 Manuele Dozzi, Federico Costantini and Luca Baron, 4 and 5 Talissa Dreossi, Agostino Dovier and Andrea Formisano.

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# Empowering Emotional Behavior Trees with Neural Computation for Digital Forensic

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**Abstract:** Empowering interactive agents with empathetic capabilities leads, on the human side, to more trust and increased engagement. This article focuses on modeling the empathetic behavior of virtual agents interacting with humans employing behavior trees which we further enhance with neuro-symbolic capabilities. This allows the specification of various kinds of empathy for the agents to be deployed for user support in critical fields such as digital forensics. This paper is a follow-up to our work in the COST Action DigForASP.

**Keywords:** Affective Computing · Human-Agent teaming · Neuro-Symbolic · Behavior Trees · Digital Forensic

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# Legal and technical challenges of AI in the field of Criminal investigations

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**Abstract:** This article addresses critical challenges in the intersection of artificial intelligence, crime investigation and digital forensics, particularly in light of the proposed Regulation on Artificial Intelligence in the European Union. The focus is on mitigating biases caused by databases and algorithms, with real-world examples highlighting discriminatory biases in criminal proceedings. The article emphasises the necessity of addressing biases in both data and algorithmic decision-making to ensure fair outcomes. Another key concern explored is the lack of traceability in AI-based decisions, posing challenges to accountability and transparency, especially in the context of criminal investigations. Additionally, the article delves into the protection of private and family data in the vast datasets analysed by AI systems, referencing a legal case that underscores the potential violation of the right to a fair trial. To address this, the article proposes the development of anonymisation systems to safeguard individuals' privacy rights. The overarching theme is the need for ethical considerations and legal frameworks to guide the responsible development and deployment of AI tools in digital forensics.

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# Fingerprint Revolution: Unleashing the Potential of Modified Bacterial Memetic Evolution for a Paradigm Shift in Fingerprint Recognition and Optimization

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**Abstract:** Biometrics refers to the science of measuring and analyzing biological or behavioral characteristics to identify people. It has been used in many fields including digital forensics, identification systems, and security. The most common biometric nowadays is fingerprint, which has improved significantly in the last two decades. Several issues in fingerprint identification and authentication systems have been raised due to factors such as displacement of the finger while scanning, fingerprint rotation, major cuts, lowering the overall system efficiency. Evolutionary algorithms evolved in recent years to enhance the system performance over time. The proposed system uses a modified version of the bacterial memetic evolutionary algorithm to overcome the identification issues and to help and support forensic experts to make reliable decisions faster. The proposed system was evaluated on five different databases and the results demonstrated that the system succeeded in identifying the correct match from the first candidate in all cases among all examined databases.

**Keywords:** Biometrics · Fingerprint · Evolutionary algorithms · Bacterial algorithm · Memetic algorithm

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# Comparison of text similarity techniques for power of attorney clauses for Polish banks

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**Abstract:** Text similarity techniques allow for close, but not exactly, matching strings to be compared and extracted from bodies of text. This functionality can be considered very useful in automated processing of the documents. In this paper existing algorithms such as Cosine similarity, Levenshtein distance, pre-trained models etc. are compared and summarised. Based on the attorney clauses from the banking sector - the official formats and given template - we consider the effectuality of each of them. An algorithm selection is made not only based on the similarity score, but also the simplicity of the given solution, often considered an advantage in a highly regulated industry. Nevertheless, this study demonstrated that pre-trained models, in certain instances, exhibit a performance that is twice as effective as other techniques that are more readily explicable in a business context.

**Keywords:** Machine learning · Natural language processing · Pre-trained language models · String matching · Levenshtein distance · Cosine Similarity

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# Using Human-Computer Interaction Data for Continuous Authentication in High-stake Electronic Assessments

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**Abstract:** As electronic assessments become more prevalent in high-risk and on-line environments, ensuring the reliability of user authentication becomes increasingly important. In this research we examine the potential for using human-computer interaction (HCI) data for improving the reliability of authentication process in electronic assessments. By analyzing user interaction patterns with assessment interfaces, we try to improve the robustness of the authentication system. Our approach uses machine learning algorithms to discern subtle behavioral signals, primarily response times, to establish a user profile. This profile is then compared against the profile claimed within the primary authentication. The main characteristic of the interaction we used in this initial research is response time. The results indicate that the data collected from participants during a typical test (49 participants and 35 questions with provided answers, in the Health Management course) are not sufficient to enhance authenticity verification as proposed. Therefore, it is necessary to develop a more comprehensive profile of participants for such an approach to make sense.

**Keywords:** Human-Computer Interaction · Authenticity · High-stake Electronic Assessments

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# Belief Change: Axiomatic Characterization of KM-Erasure

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**Abstract:** One of the most important models in the literature of belief change is *update*, defined by Katsuno and Mendelzon in 1992. In their work, KM mentioned *erasure* as the counterpart change of update. However, erasure was only defined at the basic level. In this paper: (1) we complete the axiomatics of erasure and provide its relation with update via the Levi and Harper Identities, and (2) we provide a semantics in terms of possible worlds.

**Keywords:** Belief Change · Katsuno and Mendelzon Update · Erasure · Axiomatic Characterization · possible world semantics

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# Stable Reasoning, ASP and the Interrogative Model of Inquiry

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**Abstract:** In this note we explore the relation between question-answering in answer set programming and the interrogative model of scientific inquiry proposed by Jaakko Hintikka. To this end we extend the fixpoint property of propositional equilibrium models to the first order case. In this manner we obtain a rather close agreement between the interrogative model and question-answering in ASP; this may be relevant to the topic of explanatory ASP.

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# Hypergraph logic program representation versus stratified programs

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**Abstract:** Multi-adjoint normal logic programming is a general non-monotonic logic programming framework, which makes it ideal for modeling complex scenarios. Hypergraph representation has been proved to be an appropriate tool in the study of different properties of a logic program, whereas the use of a stratification has provided interesting results related to the existence and unicity of stable models. In this paper, we will see the relation between the p-condensation graph of a program and its “optimal” stratification.

**Keywords:** Logic Programming · Hypergraphs · Negation operator

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# A preliminary taxonomy of explanations in problem solving

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**Abstract:** An important subarea of Knowledge Representation and Reasoning (KRR) in Artificial Intelligence is the field of (model-based) Declarative Problem solving. In that context, we start from some real world problem and state its premises in terms of some KRR language, so that models of that language keep a correspondence to solutions of the original problem. This is, for instance, the usual methodology followed in *Answer Set Programming* (ASP) [2], a paradigm for practical KRR typically used for problem solving in a wide range of domains and applications [4]. Explanations become crucial for the final persons that are expected to use the solutions obtained from our ASP encoding in a real world scenario. Expressing the results obtained in the answer sets in terms of the target user becomes an important requisite, and this was explored in different applications or systems for explainability in ASP [5,3,1]. However, explanations in these systems only cover some types of questions a final user may perform, but not other topics that have been studied in more technical approaches to ASP explainability. For instance, user oriented ASP systems do not normally deal with questions of the form “why does this problem have no solution?”.

In this extended abstract, we consider a preliminary taxonomy of the type of explanations that seem more common in these situations. To this aim, we use as a guide the stable roommate problem [6] where we must decide pairs of students to be assigned to double rooms in a university accommodation system according to some input preferences. Suppose we start obtaining a given solution that is presented to the user, perhaps a university administrative or a student.

The first distinction is between questions of the form *why* (positive or *real*) versus form *why-not* (negative or *alternative*). In a *why* context, the question “why  $X$  matches  $Y$ ?” is read as “how come  $X$  matches  $Y$ ?” and the expected answer is a justification that allowed us to perform such a matching: for instance  $X$  preferred  $Y$  and vice versa. On the other hand, a *why-not* asks about an alternative scenario: “why does  $X$  not match  $Z$ ?” can be read as, “could we get a matching of  $X$  and  $Z$  instead?”. Note the difference between a positive “why  $p$ ?” where we ask why  $p$  was derived in the current solution and a negative “why not  $\neg p$ ?” where we ask about another solution in which  $p$  could be false.

Second, consider again the question “why does  $X$  not match  $Z$ ?” and suppose that, indeed, there are solutions in which  $X$  matches  $Z$ . It is then reasonable to display one of those solutions, but also, to look for one *as close as possible* to the original one presented to the user. Moreover, it may be convenient to only display the main differences: for instance, if having 1000 students, the new solution only involves the reassignment of 4 of them, we would not want to see again the rest of 996 repeated assignments (we might not even notice the relevant changes among them!).



Third, suppose that instead, there is *no solution* where  $X$  matches  $Z$  at all. We could perhaps consider *hypothetical scenarios* where something exceptional is allowed. These exceptional situations could include relaxing some of the constraints (like allowing some exceptional matches), adding or deleting preferences, or even modifying the room configurations. In that way, one example of explanation could have the form “ $X$  cannot match  $Z$ , but they could if  $Y$  had not preferred  $Z$  or if we had one more extra room”. We may call this kind of explanation a “*repair*” in the sense that we imagine a hypothetical change that removes the unsolvability. If the repair is obtained from an initial solution and a *why-not* query, it makes sense, again, to include in the explanation only the relevant changes with respect to the initial solution. For instance, we have 1000 assigned students, and we ask “why  $X$  does not match  $Z$ ?”, if the repair means only a few changes, we do not want to see the rest of unchanged information.

Finally, if no repair is possible (or the user does not allow such a possibility) then the explanation can focus on the *source of unsolvability*. This may mean displaying a minimal subset of rules (or premises) that are causing the lack of solution or using other techniques to display which parts of a given scenario are involved in the unsatisfiability.

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# Efficiency of decision rule sets in classification problems

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**Abstract:** Decision algorithms [4] are considered in Rough Set Theory in order to extract non-redundant and exhaustive information from relational datasets in terms of decision rules. These algorithms are analyzed by using the notion of efficiency [5], which provides their classification quality through a value in the unit interval. Recently, we have introduced the notion of decision algorithm and efficiency in Fuzzy Rough Set Theory [1,2,3]. The first one to model and obtain the most relevant information from the dataset, and the second one to determine the suitability of the considered algorithm. However, it can be difficult to interpret the fuzzy notion of efficiency because it takes values greater than one, unlike the classical efficiency. In order to overcome this drawback, this work introduces a normalized efficiency, whose values belong to the unit interval. Moreover, the normalized efficiency is defined from a new relevance indicator of strength of decision rules, which highlights the most supported rules in the algorithm. Finally, some properties satisfied by the normalized efficiency are presented, which facilitate its interpretation.

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*Keynote speech:*

## Sensitivity Analysis as a Method for Explaining AI (XAI) in the Artificial Neural Networks Domain

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**Abstract:** The field of Explainable Artificial Intelligence (XAI) has gained prominence as the adoption of complex models, such as Artificial Neural Networks (ANNs), continues to grow. Sensitivity Analysis emerges as a pivotal method within the XAI framework, offering a systematic approach to unravelling the intricate inner workings of ANNs. This invited talk delves into the significance of Sensitivity Analysis as a method for explaining AI in the domain of Artificial Neural Networks. The presentation explores how sensitivity analysis techniques contribute to transparency, interpretability, and trust in AI systems, shedding light on the factors that influence model predictions. Through illustrative examples and case studies, the talk aims to provide valuable insights into the practical application of sensitivity analysis, bridging the gap between the complex nature of ANNs and the need for comprehensible AI systems. The talk will showcase both global and local approaches of Sensitivity Analysis, providing a comprehensive understanding of how these methods can be employed to dissect the decision-making processes within ANNs. Attendees will gain insights into the application of Sensitivity Analysis to various types of neural networks, including Multilayer Perceptrons (MLPs), Probabilistic Neural Networks (PNNs), and Convolutional Neural Networks (CNNs). Through illustrative examples and case studies, the presentation aims to demonstrate the versatility of sensitivity analysis in dissecting the complex structures of different neural network architectures. By focusing on specific neural network models, namely MLPs, PNNs, and CNNs, the talk will highlight the adaptability of sensitivity analysis across diverse AI applications. The presentation underlines a deeper understanding of how global and local sensitivity analysis techniques can enhance the interpretability and explainability of ANNs, contributing to the responsible deployment of AI systems.

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# Enhancement of Discrete Bacterial Memetic Evolutionary Algorithm for Solving The Travelling Repairman Problem

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**Abstract:** The Traveling Repairman Problem (TRP) is concerned with repairing a set of locations rather than visiting them. In this paper, we propose an enhanced version of the Discrete Bacterial Memetic Evolutionary Algorithm (DBMEA) to solve TRP. DBMEA is combining with a new method for generating the initial individual candidate solution which is called Circle Group Heuristic (CGH). CGH is constructed with the help of Genetic Algorithm (GA). The enhanced version of DBMEA with CGH has been tested for several benchmark reference data of TRP. The results show that the enhanced version has a faster and better solutions for most cases in comparison to state-of-the-art heuristics mentioned in the literature. Furthermore, for larger benchmark instances, it provided better solutions than the previously best-known results. These test results support the claim that the DBMEA with CGH is the most effective approach and recommend its use for the Traveling Repairman Problem, particularly for large instances.

**Keywords:** TRP · CGH · DBMEA · Memetic algorithm.

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# Simulated Annealing and Bacterial Foraging for Probabilistic Neural Network parameters adjustment

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**Abstract:** Probabilistic Neural Networks (PNNs), a category of Feedforward Neural Networks, leverage Kernel Density Estimators (KDEs) and the Bayesian conditional probability theorem for estimating conditional probabilities. Initially designed for classification, these networks exhibit commendable performance in both classification and regression tasks. The training process involves determining optimal or suboptimal values for the KDE smoothing parameter, commonly accomplished through analytical methods such as the Plug-in technique. Additionally, metaheuristic approaches like Particle Swarm Optimisation and Krill Herd Algorithm have been employed for smoothing parameter optimisation in PNNs due to the absence of gradient calculations. This contribution proposes the integration of Bacterial Foraging Optimisation (BFO) and Simulated Annealing (SA) for enhancing PNNs. The efficiency of these techniques in optimising PNNs is compared with the conventional Plug-in method, employing benchmark classification datasets sourced from UCI and Kaggle repositories. The results reveal that SA surpasses other methods in specific benchmarking tasks, suggesting its efficacy in training PNNs for specific problem domains.

**Keywords:** Probabilistic Neural Networks · Metaheuristics · Simulated Annealing · Bacterial Foraging Optimisation

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# Extreme Learning Machine as a New Learning Paradigm: Pros and Cons

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**Abstract:** We analyze the validity of the Extreme Learning Machine principles proposed in [2] as a new learning methodology for Single Layer Feedforward Neural Network. We show that despite the empirical success of ELM, its theoretical platform does not have a rigorous mathematical justification. To do this, we show that two main statements in [2] do not have correct proofs and are in fact incorrect. Moreover, we create a dataset that provides a counterexample to the theoretical assertions done in [2] about the ELM learning algorithm.

**Keywords:** Extreme Learning Machine · Single Layer Feedforward Neural Network · Activation function

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# Inheritance of completeness between systems of strong and weak implications

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**Abstract:** The study of unknown information in formal contexts can be done from two extremely different points of view: working just with the information available at the moment, or exploring all the different values that the unknown information can take. From these two perspectives, we obtain two kinds of attribute implications: the weak ones which are the attribute implications that hold with the current amount of information, and the strong ones which will also hold under any update of the context. We study whether, given a complete system of weak implications concerning partial formal context, one can extract a complete system of strong ones concerning the same partial formal context.

**Keywords:** Formal Concept Analysis · Unknown information · Attribute implication · Completeness

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# Connections between attribute implications in heterogeneous formal contexts and GUHA association rules

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**Abstract:** Formal concept analysis [1] is a powerful method of data analysis based on object-attribute relational data. This method outputs hierarchically ordered objects and attributes' bi-clusters based on lattice theory. In Formal concept analysis, several important generalizations regarding fuzzy sets and fuzzy logic were investigated [2,3,4,5,6,7,8,9,10,11,12,13]. Recently, we proposed extensions working with formal contexts of heterogeneous data structures [14,15,16,17]. Attribute implications can be seen as the expressions describing particular dependencies among attributes in relational data. The dependencies between attributes in a formal context were thoroughly investigated in [19,20,21]. Moreover, association rules (an extension of attribute implications) express the probability of relationships between data items within large data sets in different databases. One of the most popular association rule generation algorithms is the Apriori [22]. General Unary Hypotheses Automaton (GUHA) [23] provides association rules that extend the Apriori algorithm in several ways. In this paper, we explore the connections between attribute implications in heterogeneous formal contexts, association rules, and their extension provided by GUHA. In particular, we defined attribute implications in heterogeneous formal contexts, which allow us to apply different structure of values for each attribute. We explored the properties of attribute implications in heterogeneous formal contexts. Moreover, we explored the possibility of incorporating several types of quantifiers from GUHA association rules into fuzzy extensions of Formal concept analysis.

**Keywords:** Formal Concept Analysis · Heterogeneous formal context · Attribute implications · Association rules

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# Migrative properties for triangular conorms and fuzzy implications

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**Abstract:** Migrative and cross-migrative properties of t-norms have widely been analyzed in the literature. However, few works exist on migrative properties on t-conorms over fuzzy implications. This paper presents different features and remarks of these properties, which also justify the interconnected relationship between them.

**Keywords:** Migrativity · cross-migrativity · t-conorm · fuzzy implication

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# Masonry strength assessment based on Fuzzy signature model

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**Abstract:** Fuzzy signatures have been successfully used for various engineering applications including the strength and condition assessment of structural materials. In this paper, fuzzy signatures were used to determine the compressive strength of masonry based on groups of related measured values. Handling the uncertainty this way seemed useful because of the subjective parts and influencing noise factors of the measurements which are incorporated as leaves in the signature. For such modeling the structure from where the algebraic framework can be obtained which allows making computations with the fuzzy signatures thus determined. Since multiplicative type aggregations are applied on the various material test results assigned to the leaves of the signatures for the determination of the compressive strength of masonry, fuzzy arithmetic multiplication based on Zadeh's extension principle was applied here. To perform the fuzzy signature calculations, the scalar product and the  $n$ -th root of fuzzy numbers were used.

**Keywords:** Fuzzy signatures · Weighted geometric aggregations · Compressive strength · Masonry.

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